CDO models: Opening the black box – Part two The Finite Homogenous Pool model

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Structured credit research: Global

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CDO models: Opening the black box - Part two

Putting the theory to work...

The Finite Homogeneous Pool model

- After having released the Large Homogenous Pool Model in the first part of our series, we now move towards the finite homogenous pool model
- This publication builds up on the first publication of this series (<u>CDO models: Opening the black box</u>) and has been structured as a guide to be used in conjunction with the excel-based Finite Homogenous Pool model. Whilst we do briefly touch upon the main theoretical concepts, we do not go into detailed explanations and proofs, as this has been widely discussed and is readily available. Instead, we focus on how to implement the theory and apply the models
- We start by taking a closer look at how the loss distribution is constructed for a finite pool of homogenous assets
- Afterwards we address some important factors that have to be considered when using the Gauss-Hermite integration technique in combination with the binomial distribution

Finite Homogeneous Pool Model:

https://research.dresdnerkleinwort.com/document/FILE.pdf?REF=244439



Just like tossing a coin..

The Binomial distribution is used to build up the default distribution

- > The assumption of homogenous assets allows us to use the Binomial distribution to construct the default distribution
- > The Binomial distribution is a discrete distribution used to model binary random variables (eg "tossing a coin")
- For a given number of trials *n*, the probability of *k* successful trials (observing exactly *k*=20 tails for *n*=40 coin tosses) is given by:

$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

- ▶ Where *p* denotes the likelihood to succeed in a given attempt (50% for a fair coin)
- Within the context of the finite homogenous pool model, we can therefore apply this distribution by parametrising
 - > n = number of assets
 - \succ *k* = number of defaults,
 - \rightarrow p = PD(t|m), the probability of default by time t conditional on the common factor m

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From conditional to unconditional defaults

The Binomial distribution is used to build up the default distribution

As we assume homogeneity, the PDs are identical for all assets and as before are given by

$$PD(t \mid m) = \Phi\left(\frac{\text{Default Barrier}(t) - \sqrt{\rho m}}{\sqrt{1 - \rho}}\right)$$

Applying the Binomial distribution, the probability of observing excatly k defaults conditional on the realisation of a common factor m is therefore given by

Prob('k defaults until t'|m) =
$$\binom{n}{k} PD(t | m)^k (1 - PD(t | m))^{n-k}$$

► The unconditional default distribution until time *t* is then given by

Prob ('up to k defaults by time t') =
$$\sum_{n=0}^{k} \binom{n}{k} \int_{-\infty}^{+\infty} \left(\Phi \left(\frac{\text{Default Barrier}(t) - \sqrt{\rho m}}{\sqrt{1 - \rho}} \right) \right)^{k} \left(1 - \Phi \left(\frac{\text{Default Barrier}(t) - \sqrt{\rho m}}{\sqrt{1 - \rho}} \right) \right)^{n-k} \phi(m) dm$$

- We integrate over the common factor to receive unconditional probabilities, for *k* defaults
- Then we sum over all defaults up to k (0,1,...,k-1,k) to calculate the cumulative default distribution until time t



Our spreadsheet - sheet "integration"







Building the loss distribution – sheet "integration"

Screenshot (2/5) (spread 100 bps, 40% rec., 20% correlation)



For each iteration step, the macro copies the unconditional probabilities of default to the matrix below.

Once completed, the portfolio's expected default rate as well as the default distribution can be obtained easily

48% 56%

Default Rate

64% 72% 80% 88%

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96%

Loss and recovery waterfalls - sheet "pricing"

Screenshot (3/5) (spread 100 bps, 40% rec., 20% correlation)





Tranche pricing

From conditional to unconditional tranche loss and recovery

- > The way the calculation of the tranche losses and recoveries is implemented differs slightly from the previously published LHP models
- We now directly multiply the tranche's principal loss for a given number of defaults (ie a specific point of the loss distribution) with the corresponding unconditional probability of observing precisely this event
- The cumulative loss is therefore calculated by multiplying the default probability matrix (shown on slide 5 in pink) with the tranche's principal loss (shown on slide 6)
- The cumulative recoveries are obtained analogously. The default probability matrix is now multiplied with the tranche's recovered amount (shown on slide 6)

Screenshot (4/5) (spread 100 bps, 40% rec., 20% correlation)



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Tranche pricing: equivalent to pricing single name CDS

Screenshot (5/5) (spread 100 bps, 40% rec., 20% correlation)

Once the loss distribution has been built and the losses and recoveries for each tranche have been determined, tranche pricing is straightforward and implemented identical to the previous models





The Gauss-Hermite integration technique (1/2)

A numerical procedure to increase speed

- ► A variant of the Gaussian quadrature, the Gauss Hermite integration can be used to approximate integrals ranging from (-∞,+∞)
- As demonstrated in the our first two spreadsheets for the Large homogenous pool model (LHPM), the basic idea is to reduce the number of points that are used to approximate the distribution of the common factor
- While in the basic version of the LHPM we integrated over the interval (-5, 5) using decimal steps, resulting in 101 points, in the LHPM with the Gaussian-Hermite we reduced this to 30 points
- While significantly reducing the number of calculations that have to be carried out and therefore increasing speed, we still achieve a high level of accuracy because of the way the points are chosen within the Gaussian Hermite integration approach
- The 30 points we use in model two as well as in this model achieve almost the same level of accuracy as the 101 equally spaced points in model one, and a considerably higher level of precision than would be achieved with 30 equally spaced points



The Gauss-Hermite integration technique – some caveats (2/2)

A numerical procedure to increase speed

- However, as with any approximation, the gain in speed comes at a cost
- For a low number of points for the Gauss-Hermite, especially in combination with high spreads and correlations, the loss distribution will exhibit waves. The higher the number of assets, the more pronounced this behaviour
- These waves do however not result from the use of the Gaussian-Hermite technique per se but are rather induced by the fact that we now look at a finite rather than infinite pool of homogenous assets
- For given model inputs with respect to number of assets, spreads, recoveries and correlations, increasing the number of points results in a smoother default distribution.
- As this comes at the cost of slower computation, when choosing the adequate number of points for the Gauss-Hermite, the trade-off between calculation speed and the required degree of accuracy has to be kept in mind





Disclosure appendix

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lssuer	Definition
Overweight	We expect the issuer to outperform sector peers over a 6-months horizon and would suggest holding more of the issuer's instruments than the market would hold on average. The recommendation reflects our weighted view on all of an issuer's instruments and fundamentals compared to sector peers
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